WAEC/NECO/GCE REPEATED TOPICS IN MATHEMATICS

$y^2 = y \times y$

$x^5 = x \times x \times x \times x \times x$

Topic 1. Indices (Exponents)

An exponent tells you how many times to multiply a number (the base) by itself.

Example

- 2^3 means $2 \times 2 \times 2 = 8$
- 5^2 means $5 \times 5 = 25$

Rules for working with powers.

Laws of Indices

1. Multiplication Law

$a^m \times a^n = a^{m+n}$

2. Division Law

$a^m \div a^n = a^{m-n}$

3. Power Law

$(a^m)^n = a^{mn}$

4. Zero Law

$2^3 \times 2^4 = 2\times2\times2\times2\times2\times2\times2 = 2^7$

$= 2^{3+4} = 2^7$

$10^5 \times 10^1 = 10^{5+1} = 10^6$

$10^7 \div 10^3 = \frac{10\times10\times10\times10\times10\times10\times10}{10\times10\times10} = 10^4$

$= 10^{7-3} = 10^4$

$x^3 \div x^2 = \frac{x\times x\times x}{x\times x} = x^1$

$x^{3-2} = x^1$

$(2^3)^2 = 2^{3\times2} = 2^6$

$(10^5)^3 = 10^{5\times3} = 10^{15}$

$10^0 = 1, \; y^0 = 1$

$a^0 = 1$

5. Fractional Index Law

$a^{-n} = 1/a^n$

$a^{-n} = \frac{1}{a^n}$

$10^{-3} = \frac{1}{10^3}$

$5^{-2} = \frac{1}{5^2}$

b. Standard Form (Scientific Notation)

Writing numbers as $a \times 10^n$, where $1 \leq a <$ 10 and n is an integer.

- Example: $4500 = 4.5 \times 10^3$

- Example: $0.0032 = 3.2 \times 10^{-3}$

$a \geq 1$ $a = 1, 2, 3, 4, 5, 6, 7, 8, 9$

$a < 10$

$4500 = 4.5 \times 10^3$

$0.00035 = 3.5 \times 10^{-4}$

$a^k = N$

$2^3 = 8$

then $\log_2 8 = 3$

$5^2 = 25$

then $\log_5 25 = 2$

$6^2 = 36$

then $\log_6 36 = 2$

Topic 2. Logarithms

Logarithms are the inverse of indices (exponents).

a. Relationship Between Indices and Logarithms

If $a^k = x$, then $\log_a x = k$

- Example: $10^2 = 100$ implies $\log_{10} 100 = 2$

- Example: $y = 10^k$ implies $\log_{10} y = k$

b. Basic Rules of Logarithms

1. $\log_a (xy) = \log_a x + \log_a y$
2. $\log_a (x/y) = \log_a x - \log_a y$
3. $\log_a x^n = n \log_a x$
4. $\log_a a = 1$
5. $\log_a 1 = 0$

$a^m \cdot a^n = a^{m+n}$

$\log_a(xy) = \log_a x + \log_a y$

$\log_{10}(10 \times 5) = \log_{10} 10 + \log_{10} 5$

$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$

$\log_{10}\left(\frac{100}{10}\right) = \log_{10} 100 - \log_{10} 10$

$\log_a x^n = n \log_a x$

$\log_{10} 10^3 = 3 \log_{10} 10$

$\log_a a = 1, \log_{10} 10 = 1$

$\log_2 2 = 1 \quad \log_5 5^2 = 1$

$\log_3 3 = 1$

$\log_a 1 = 0$

$\log_{10} 1 = 0$

$\log_2 1 = 0$

c. Using Log Tables and Antilogs

- Log tables give log values; antilogs give numbers from logs.

- Example: log 4.5 ≈ 0.6532 → antilog(0.6532) ≈ 4.5

log value of 4.5

d. Calculations

- Multiplication: $\log(xy) = \log x + \log y$
- Division: $\log(x/y) = \log x - \log y$
- Powers: $\log(x^n) = n \log x$
- Roots: $\log(\sqrt[n]{x}) = (1/n) \log x$

Topic 3. Surds (Radicals)

Surds are numbers involving roots (like √). $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{6}, \sqrt{7}$

Surds are numbers expressed with roots (like √, $\sqrt[3]{}$) that can't be simplified to whole numbers.

Examples

- $\sqrt{2}$ (can't be simplified)

- $\sqrt{4} = 2$ (not a surd, it's a whole number)
- $3 + \sqrt{5}$ (a surd)

$\sqrt{108} = \sqrt{9 \times 12}$
$= \sqrt{9} \times \sqrt{12}$
$= \sqrt{9} \times \sqrt{4} \times \sqrt{3}$
$= 3 \times 2 \times \sqrt{3}$
$= 6\sqrt{3}$

$\sqrt{300} = \sqrt{100 \times 3}$
$= 10 \times \sqrt{3} = 10\sqrt{3}$

b. Simplification

- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ $= \sqrt{3 \times 9} = \sqrt{3} \times \sqrt{9} = 3\sqrt{3}$
- $\sqrt{(a/b)} = \sqrt{a} / \sqrt{b}$ $= \sqrt{\frac{16}{4}} = \frac{\sqrt{16}}{\sqrt{4}} = \frac{4}{2} = 2$
- Example: $\sqrt{12} = \sqrt{(4 \times 3)} = 2\sqrt{3}$

$\sqrt{4} \times \sqrt{3} = 2\sqrt{3}$

c. Rationalizing

- Remove root from denominator: multiply top/bottom by root
- Example: $1/\sqrt{2} = (1\times\sqrt{2})/(\sqrt{2}\times\sqrt{2}) = \sqrt{2}/2$

$\sqrt{3*3} = \sqrt{9} = 3$

$\sqrt{a}*\sqrt{a} = a$

$3+\sqrt{5}$ = conjugate $3-\sqrt{5}$

$\frac{3}{\sqrt{3}} * \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{3}$

$\frac{3}{3+\sqrt{5}} = \frac{3}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}} = \frac{3(3-\sqrt{5})}{9+3\sqrt{5}-3\sqrt{5}-5} = \frac{9-3\sqrt{5}}{4}$

$a\sqrt{b} \pm c\sqrt{b} = a \pm c(\sqrt{b})$

$2\sqrt{3} + 5\sqrt{3} = 2+5(\sqrt{3})$

$= 7\sqrt{3}$

$\sqrt{27} + \sqrt{300}$

$\sqrt{27} = \sqrt{9 \times 3} = 3\sqrt{3}$

$\sqrt{300} = \sqrt{100 \times 3} = 10\sqrt{3}$

$3\sqrt{3} + 10\sqrt{3} = 13\sqrt{3}$

d. Basic Operations

- Addition and Subtraction of Surds

$a\sqrt{b} \pm c\sqrt{b} = (a \pm c)\sqrt{b}$

- Multiplication of Surds

$\sqrt{a} \times \sqrt{b} = \sqrt{(ab)}$

Examples

- Simplification of Surds : $\sqrt{8} = 2\sqrt{2}$

$= \sqrt{8} = \sqrt{4 \times 2} = 2\sqrt{2}$

- Rationalizaton of Surds : $1/(3-\sqrt{2})$

rationalize: multiply by $(3+\sqrt{2})$

$\frac{1}{3-\sqrt{2}} = \frac{1}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}}$

$\frac{1(3+\sqrt{2})}{9+3\sqrt{2}-3\sqrt{2}-2} = \frac{3+\sqrt{2}}{7}$

- $\sqrt{3} + 2\sqrt{3} = 3\sqrt{3}$

$$S.I = \frac{P \times R \times T}{100}$$

Topic 4. Percentages

1. **Simple Interest:** Simple interest is interest calculated only on the original amount (principal) borrowed or invested.

Formula

- Simple Interest (I) = (P × R × T)/100
- P = Principal
- R = Rate (% per year)

- T = Time (years)

Example

- ₦1000 at 5% for 2 years → I = (1000×5×2)/100 = ₦100

- Amount = P + I

2. **Commission:** Commission is a fee paid to someone for making a sale or

P = 150,000
R = 5%
T = 2years
I = 150,000 × 5 × 2 / 100
= 1500 × 5 × 2
= 1500 × 10 = 15000
Amount = Int + Pri
= 15000 + 150000
= 165,000

Comm = 10/100 × 50,000

= 500 × 10

= 5000

completing a task, usually a % of the amount involved.

Example

- You sell ₦5000 worth of goods with 10% commission → You get ₦500.

- Commission = % × Amount

- Example: 10% of ₦5000 = ₦500

3. **Discount: Discount is a reduction in price, often a % off the original price.**

Example

- ₦1000 item with 20% discount → Pay ₦800.

- Discount = % × Price

- Selling Price = Original - Discount

200,000 — discount 10%

10/100 × 200,000

200000 × 10 = 20,000

200000 − 20,000 = 180,000

S.P = Orig − disc

10/100 × 500000/1

10% × 500000 = 50,000

1st = 500000 − 50,000
= 450,000

2yrs = 450000 − 45000
= 405,000

4. **Depreciation: Depreciation is when something loses value over time.**

Example

- A ₦500,000 car depreciates 10% yearly →

Worth ₦450,000 after 1 year.

450000
45000
405,000

- Value decreases over time

5. **Profit & Loss:**

- Profit: You sell something for more than you bought it.

- Loss: You sell for less than you bought it.

- Profit = SP - CP

- Loss = CP - SP

Examples

- Buy for ₦500, sell for ₦700 → Profit ₦200.
- Buy for ₦500, sell for ₦300 → Loss ₦200.

6. **Compound Interest:** Compound interest is interest calculated on the original amount PLUS any interest already added.

- $A = P(1 + r/100)^n$

Profit = SP − CP
700 − 500 = 200

Loss = CP − SP
= 500 − 300 = 200

% Profit = $\frac{\text{Profit}}{CP} \times \frac{100}{1}$

% Loss = $\frac{\text{Loss}}{CP} \times \frac{100}{1}$

% = $\frac{200}{500} \times \frac{100}{1} = \frac{200}{5} = 40\%$

% Loss = $\frac{200}{500} \times \frac{100}{1}$

$= \frac{200}{5} = 40\%$

$A = P\left(1 + \frac{r}{100}\right)^n$

- CI = A - P

Example

- ₦1000 at 10% yearly:
  - Year 1: ₦100 interest → ₦1100
  - Year 2: 10% of ₦1100 = ₦110 interest → ₦1210

7. **Hire Purchase:** Hire purchase is paying for something in instalment, with interest, until you own it.

5000×12 = 60,000

Example

- Buy a ₦50,000 phone → Pay ₦5,000 monthly for 12 months (with interest).

- Pay in instalments + interest

8. **Percentage Error:** Percentage error shows how accurate a measurement is, compared to the actual value.

Formula

- (|Actual - Measured| / Actual) × 100%

Example

- Actual = 100, Measured = 90

Error = (|100-90|/100) × 100% = 10%

$\frac{\text{Error}}{\text{Actual}} \times \frac{100}{1} = \%\,\text{Error}$

Actu. = 100 m

Measured = 95 m

Error = 100 − 95 = 5 m

$\%\,\text{error} = \frac{5}{100} \times \frac{100}{1}$

$= 5\%$

Topic 5. **Simple operations on algebraic expressions**

a. Expansion of Algebraic Expressions

Expansion involves multiplying terms inside brackets to simplify expressions.

Methods

1. Distributive Law: a(b + c) = ab + ac

2. FOIL Method for binomials: $(x + a)(x + b)$

$= x^2 + (a+b)x + ab$

$x^2 + xa + xb + ab$

$x^2 + x(a+b) + ab$

$(x-a)(x-b)$

$x^2 - ax - bx + ab$

$x^2 - (a+b)x + ab$

Examples

- $3(x + 4) = 3x + 12$
- $(x + 2)(x + 5) = x^2 + 7x + 10$
- $(2x - 1)(x + 3) = 2x^2 + 5x - 3$

$2x^2 + 6x - x - 3 = 2x^2 + 5x - 3$

b. Factorization of Algebraic Expressions

Factorization is finding factors that multiply to give the original expression.

Methods

1. Common Factor: Find common term in all parts

- Example: $2x + 4 = 2(x + 2)$

$2(x+2) = 2x+4$

2. Grouping: Group terms to factor

- Example: ax + ay + bx + by = (a+b)(x+y)

3. Quadratic: Find pairs that work

- Example: $x^2 + 5x + 6 = (x+2)(x+3)$